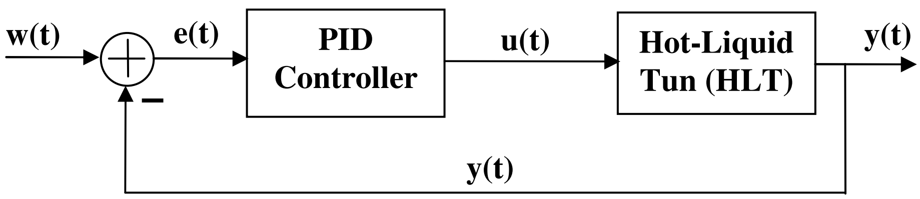
1. Introduction

This document describes the derivation of a PID controller that can be implemented in the brew application. The PID controller should be capable of controlling the temperature of the Hot Liquid Tun (HLT, 90 L) to within 0.5 °C.
The HLT contains a heating element of 3 kW, which is driven by the PID controller output signal Gamma [0..10 %]. The HLT temperature sensor is a LM92 12 bit + sign bit, ± 0.33°C accurate temperature sensor.

This document contains the following information

- Chapter 2: Derivation of a time-discrete algorithm for a PID controller
- Chapter 3: Derivation of an improved algorithm (a so-called 'type C' PID controller)
- Chapter 4: Description of algorithms for finding the optimum set of K_c, T_i, T_d and T_s values of the PID controller
- Chapter 5: Experimental results
- An appendix containing the C source listing

2. Derivation of a time-discrete algorithm for a PID controller



The generic equation[1] for a PID controller in the time-continuous domain is:

$$u(t) = K_c . \left(e(t) + \frac{1}{T_i} \int e(\tau) d\tau + T_d . \frac{de(t)}{dt} \right) \qquad eq.\ 01$$

With:

	$K_c = K_p$	Proportional Gain (for our temperature controller, unity is [% / °C])
	$T_i = K_c / K_i$	Time-constant Integral gain [sec.]
	$T_d = K_d / K_c$	Time-constant Derivative gain [sec.]
	T_s	Sample period (default value is 5 seconds)
	w(t)	Set point (SP) value for temperature. Is also called T_{set_hlt} in this document
	e(t)	error signal = set-point w(t) – process variable y(t) = $T_{set\_hlt} - T_{hlt}$
	u(t)	PID output signal, also called Gamma, ranges from [0..100 %]
	y(t)	process variable PV = measured temperature (also called T_{hlt} in this document)

The corresponding equation in the s-domain is then:

$$\frac{U(s)}{E(s)} = K_c . \left(1 + \frac{1}{T_i . s} + T_d . s \right) \qquad eq.\ 02$$

This transfer function has no real practical use, since the gain is increased as the frequency increases. Practical PID controllers limit this high frequency gain, using a first order low-pass filter. This results in the following transfer function:

[1] This is the ideal, textbook version of a continuous-time PID controller. See [1], page 54.

$$\frac{U(s)}{E(s)} = K_c.\left(1+\frac{1}{T_i.s}+\frac{T_d.s}{\gamma.s+1}\right) = K_c.\left(1+\frac{1}{T_i.s}+\frac{T_d}{\gamma+s^{-1}}\right) \qquad eq.\ 03$$

where γ is a small time-constant and may be set as 10% of the value of the derivative term T_d.

Equation eq. 03 needs to transferred to the Z domain to make it suitable for implementation on a computer. This is done using the bilinear transformation (given in eq. 04):

The bilinear transformation formula is given with: $s = \frac{2}{T_s}.\frac{1-z^{-1}}{1+z^{-1}}$ *eq. 04*

Now use the bilinear transformation, given in equation *eq. 04,* to transform equation *eq. 03* onto an equivalent form in the Z –domain:

$$\frac{U(z)}{E(z)} = K_c.\left(1+\frac{T_s}{2.T_i}.\frac{1+z^{-1}}{1-z^{-1}}+\frac{T_d.\left(1-z^{-1}\right)}{\gamma.\left(1-z^{-1}\right)+\frac{T_s}{2}.\left(1+z^{-1}\right)}\right) \qquad eq.05\text{-}1$$

This transforms into:

$$\frac{U(z)}{E(z)} = K_c.\left(1+\frac{T_s}{2.T_i}.\frac{1+z^{-1}}{1-z^{-1}}+2.T_d.\frac{1-z^{-1}}{T_s+2.\gamma+z^{-1}.(T_s-2.\gamma)}\right) \qquad eq.05\text{-}2$$

$$\frac{U(z)}{E(z)} = K_c.\left(1+\frac{T_s}{2.T_i}.\frac{1+z^{-1}}{1-z^{-1}}+\frac{2.T_d}{T_s+2.\gamma}.\frac{1-z^{-1}}{1+z^{-1}.\frac{T_s-2.\gamma}{T_s+2.\gamma}}\right) \qquad eq.05\text{-}3$$

Now, let all separate parts share the same denominator:

$$\frac{U(z)}{E(z)} = K_c.\left(\frac{\left(1-z^{-1}\right).\left(1+z^{-1}.\frac{T_s-2\gamma}{T_s+2\gamma}\right)+\frac{T_s}{2.T_i}.\left(1+z^{-1}\right).\left(1+z^{-1}.\frac{T_s-2\gamma}{T_s+2\gamma}\right)+\frac{2T_d}{T_s+2\gamma}.\left(1-z^{-1}\right)^2}{\left(1-z^{-1}\right).\left(1+z^{-1}.\frac{T_s-2\gamma}{T_s+2\gamma}\right)}\right) \qquad eq.05\text{-}4$$

Rewrite equation eq.05-4 and combine all parts of z^{-1} and z^{-2} with each other:

$$\frac{U(z)}{E(z)} = K_c.\left(\frac{1+z^{-1}.\frac{T_s-2\gamma}{T_s+2\gamma}-z^{-1}-z^{-2}.\frac{T_s-2\gamma}{T_s+2\gamma}+\frac{T_s}{2T_i}.\left(1+z^{-1}.\frac{T_s-2\gamma}{T_s+2\gamma}+z^{-1}+z^{-2}.\frac{T_s-2\gamma}{T_s+2\gamma}\right)+\frac{2T_d}{T_s+2\gamma}.\left(1-2z^{-1}+z^{-2}\right)}{1+z^{-1}.\frac{T_s-2.\gamma}{T_s+2.\gamma}-z^{-1}-z^{-2}.\frac{T_s-2.\gamma}{T_s+2.\gamma}}\right)$$

... *eq.05-5*

Simplifying the various terms results in:

$$\frac{U(z)}{E(z)} = K_c . \left(\frac{1 + \frac{T_s}{2T_i} + \frac{2T_d}{T_s + 2\gamma} + z^{-1} . \left(\frac{\frac{T_s^2}{T_i} - 4\gamma - 4T_d}{T_s + 2\gamma} \right) + z^{-2} . \frac{\left[2\gamma - T_s + \frac{T_s.(T_s - 2\gamma)}{2.T_i} + 2T_d \right]}{T_s + 2\gamma}}{1 - z^{-1} . \frac{4\gamma}{T_s + 2\gamma} - z^{-2} . \frac{T_s - 2\gamma}{T_s + 2\gamma}} \right) \qquad \textit{(eq.05-6)}$$

Now define the following parameters:

$$k_0 = K_c . \left(1 + \frac{T_s}{2T_i} + \frac{2T_d}{T_s + 2\gamma} \right) \quad ; \quad k_1 = K_c . \left(\frac{\frac{T_s^2}{T_i} - 4\gamma - 4T_d}{T_s + 2\gamma} \right)$$

$$k_2 = K_c . \left(\frac{2.\gamma - T_s + \frac{T_s^2}{2T_i} - \frac{\gamma.T_s}{T_i} + 2T_d}{T_s + 2\gamma} \right) \quad ; \quad p_1 = \frac{4\gamma}{T_s + 2\gamma} \quad ; \quad p_2 = \frac{T_s - 2\gamma}{T_s + 2\gamma} \qquad \textit{(eq.06)}$$

Substituting these parameters back into equation 05-6 results in:

$$U(z).\left(1 - p_1.z^{-1} - p_2.z^{-2}\right) = E(z).\left(k_0 + k_1.z^{-1} + k_2.z^{-2}\right) \qquad \textit{(eq. 07)}$$

Transforming equation *eq. 07* back to the time-discrete form results in:

$$u[k] = p_1.u[k-1] + p_2.u[k-2] + k_0.e[k] + k_1.e[k-1] + k_2.e[k-2] \qquad \textit{(eq. 08)}$$

Equation eq.08 is implemented with **pid_reg2()** and eq.07 is implemented with **init_pid2()** (see appendix for full C source listing).

3. Derivation of a Type C PID controller

There are three types of PID equations (see http://bestune.50megs.com/typeabc.htm) , with type C being the preferred one. Equation eq. 01 (and ultimately eq. 08) are type A equations, since the P- and the D-term both contain the set-point. Any changes in the set-point may cause an unwanted change in the PID output u(t).
Removing the set-point from the D-term results in a type B controller. The type C controller has also removed the set-point from the P-term, resulting in an even better PID controller implementation.

Starting with equation 01 and differentiating both sides gives equation eq. 09

$$du(t) = K_c.\left(de(t) + \frac{e(t)}{T_i} + T_d.\frac{d^2e(t)}{dt}\right) \qquad (eq.\ 09)$$

Transforming equation eq. 09 to the time-discrete domain, using backwards differentiation, results in equation eq. 10:

$$u_k = u_{k-1} + K_c.\left[(e_k - e_{k-1}) + \frac{T_s.e_k}{T_i} + \frac{T_d}{T_s}.(e_k - 2.e_{k-1} + e_{k-2})\right] \qquad (eq.\ 10)$$

The D-term needs to be filtered with a Low-Pass Filter (LPF) to make it more practical. The transfer function of a simple LPF is given with:

$H(s) = \dfrac{1}{\gamma.s + 1}$, with γ typically set to about 10% of the T_d value. $(eq.11a)$

The equivalent Z transfer function is:

$$H(z) = \frac{T_s.(1+z^{-1})}{2.\gamma.(1-z^{-1}) + T_s.(1+z^{-1})} = \frac{T_s}{T_s + 2.\gamma}.\frac{1+z^{-1}}{1 + \frac{T_s - 2.\gamma}{T_s + 2.\gamma}.z^{-1}} \qquad (eq.11b)$$

The equivalent function is the time-discrete domain is then:

$$lpf_k = \frac{2.\gamma - T_s}{2.\gamma + T_s}.lpf_{k-1} + \frac{T_s}{T_s + 2.\gamma}.(e_k + e_{k-1}) \qquad (eq.11c)$$

Equation eq.10 can now also be written as:

$$u_k = u_{k-1} + K_c.\left[(e_k - e_{k-1}) + \frac{T_s.e_k}{T_i} + \frac{T_d}{T_s}.(lpf_k - 2.lpf_{k-1} + lpf_{k-2})\right] \qquad (eq.\ 12)$$

Equation 12 is still a type A equation (“textbook PID”), because the K_c term depends on e_k and the input of the LPF also has e_k as input. Equation eq.11c and eq.12 are implemented with **pid_reg3()** and with **init_pid3()** (see appendix for full C source listing).

Because it is not easy to transform this equation into a full type C controller (because of the addition in eq.11c), we will revert to equation eq.10 and transform this equation into a type C equation (eq. 13):

$$u_k = u_{k-1} + K_c.\left[\left(e_k - e_{k-1}\right) + \frac{T_s.e_k}{T_i} + \frac{T_d}{T_s}.\left(e_k - 2.e_{k-1} + e_{k-2}\right)\right] \qquad (eq.10)$$

$$u_k = u_{k-1} + K_c.\left[SP_k - PV_k - \left(SP_{k-1} - PV_{k-1}\right) + \frac{T_s.e_k}{T_i} + \frac{T_d}{T_s}.\left(SP_k - PV_k - 2.\left(SP_{k-1} - PV_{k-1}\right) + SP_{k-2} - PV_{k-2}\right)\right] \qquad (eq.13)$$

Here, PV is the process variable, which is T_{hlt} (the actual temperature of the HLT). Furthermore SP is the set-point or the reference temperature.

If we assume that the set-point is not changed, we can state the $PV_k = PV_{k-1} = PV_{k-2}$. With this, the equation transforms into:

$$u_k = u_{k-1} + K_c.\left[\left(PV_{k-1} - PV_k\right) + \frac{T_s.e_k}{T_i} + \frac{T_d}{T_s}.\left(2.PV_{k-1} - PV_k - PV_{k-2}\right)\right] \qquad (eq.14)$$

Equation 14 is a type C PID controller and normally referred to as a Takahaski PID controller. This equation is implemented with **pid_reg4()** and with **init_pid4()** (see appendix for full C source listing).

4. Finding the optimum set of PID parameters

Finding the optimum parameters for a PID controller can be difficult. Optimum means that the set-point temperature is reached as quickly as possible with overshoot minimised.

Three well-known algorithms for determining the PID parameters are described here:

- Ziegler-Nichols open-loop: set PID controller to a certain output and determine slope and dead-time of HLT system
- Ziegler-Nichols closed-loop: measure step-response
- Cohen-Coon: also a closed-loop method. Measure step-response
- Integral of the time weighted absolute error (ITAE): results in the best performance. The error signal is minimised (over time).

Some terms are frequently used in this document:

- **Dead-time Θ:** this is the time-delay between the initial step and a 10% increase in the process variable (the HLT temperature in our case).
- **K_{hlt}:** the gain of the HLT-system. The HLT-system receives the Gamma value (PID output) as input and has the HLT temperature as output. Unity of K_{hlt} is [°C / %].
- **τ_{hlt}:** the time-constant of the HLT-system. The HLT-system can be described with a first-order process model with time-delay (FOPTD). The transfer function for this model is:

$$HLT(s) = \frac{K_{hlt}.e^{-\theta.s}}{\tau_{hlt}.s+1}$$

- **a***: the normalised slope of the step response. Equal to ΔT / (Δt. Δp) with:
    - ΔT : change in temperature [°C]
    - Δt : change in time [seconds]
    - Δp: change in PID controller output [%]

With these three parameters, the optimum PID parameters are determined using table 1 on the next page (values are given both for PID operation and for PI-only operation):

Method: / Parameter:	**K_c [% / °C]**	**T_i [seconds]**	**T_d [seconds]**
Ziegler-Nichols Open-loop	$K_c = \frac{1,2}{\theta.a*}$	$T_i = 2,0.\theta$	$T_d = 0,5.\theta$
Ziegler-Nichols Open-loop	$K_c = \frac{0,9}{\theta.a*}$	$T_i = 3,33.\theta$	- -
Ziegler-Nichols Closed-loop	$K_c = \frac{1,2.\tau_{hlt}}{K_{hlt}.\theta}$	$T_i = 2,0.\theta$	$T_d = 0,5.\theta$
Ziegler-Nichols Closed-loop	$K_c = \frac{0,9.\tau_{hlt}}{K_{hlt}.\theta}$	$T_i = 3,33.\theta$	- -
Cohen-Coon	$K_c = \frac{\tau_{hlt}}{K_{hlt}.\theta}.\left(\frac{\theta}{4.\tau_{hlt}} + \frac{4}{3}\right)$	$T_i = \theta.\frac{32.\tau_{hlt} + 6.\theta}{13.\tau_{hlt} + 8.\theta}$	$T_d = \theta.\frac{4.\tau_{hlt}}{2.\theta + 11.\tau_{hlt}}$
Cohen-Coon	$K_c = \frac{\tau_{hlt}}{K_{hlt}.\theta}.\left(\frac{\theta}{12.\tau_{hlt}} + \frac{9}{10}\right)$	$T_i = \theta.\frac{30.\tau_{hlt} + 3.\theta}{9.\tau_{hlt} + 20.\theta}$	- -
ITAE-Load	$K_c = \frac{1,357}{K_{hlt}}.\left(\frac{\theta}{\tau_{hlt}}\right)^{-0,947}$	$T_i = \frac{\tau_{hlt}}{0,842}.\left(\frac{\theta}{\tau_{hlt}}\right)^{0,738}$	$T_d = 0,381.\tau_{hlt}.\left(\frac{\theta}{\tau_{hlt}}\right)^{0,995}$
ITAE-Load	$K_c = \frac{0,859}{K_{hlt}}.\left(\frac{\theta}{\tau_{hlt}}\right)^{-0,977}$	$T_i = \frac{\tau_{hlt}}{0,674}.\left(\frac{\theta}{\tau_{hlt}}\right)^{0,680}$	- -

Table 1: optimum PID parameters for the various methods

To be able to find these three parameters accurately, two experiments need to be conducted. These two experiments are described in the next two paragraphs.
The last paragraph (§ 4.3) shows the calculated PID parameters for all these methods.

4.1 Experiment 1: Determine dead-time Θ of HLT-system

Manually set the PID-controller output ("gamma") to a certain value (e.g. 20 %). In case of a heavy load, 100 % is recommended (more accurate), but if you don't know the performance of the system, a lower value to start with is better. The temperature starts to increase and follows the curve in figure 1.

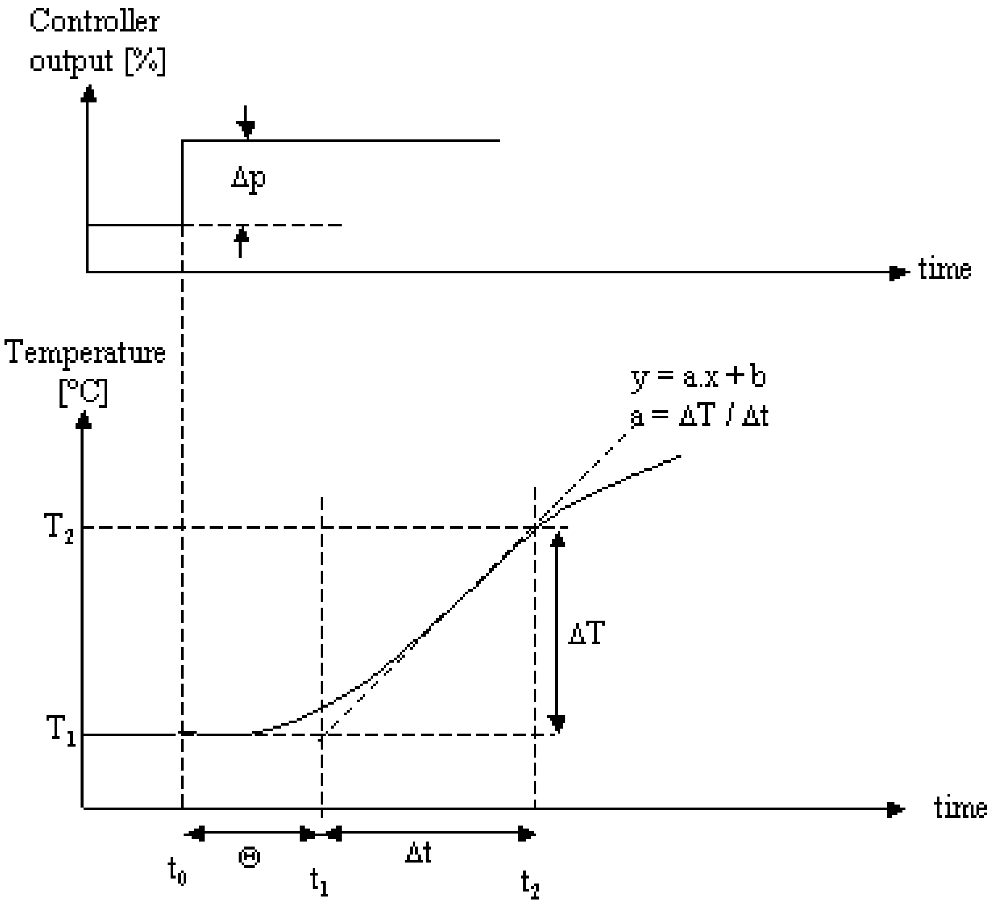


Figure 1: Open-loop response of the system

Calculate the average slope of the rise ($a = \Delta T / \Delta t$) by using regression analysis if possible.
Calculate the normalised slope a*, which is defined as: $a^* = a / \Delta p$.
The dead-time Θ is defined as the time between t_0 and t_1 (by using regression analysis, this can be done quite accurately).

<u>Experimental data:[2] HLT filled with 85 L water, lid off</u>

19:08:20	t_0, the PID controller output gamma was set to 100 %, T_1 = 47,41 °C
	Step-response up to 55.00 °C, ΔT = 7,59 °C
19 :12 :25	T_{hlt} = 48,18 °C (≥ T_1 + 10%.ΔT = 48,17 °C)
19 :27 :21	T_{hlt} = 54,25 °C (≥ T_1 + 90%.ΔT = 54,24 °C)

Regression analysis of the data between 19 :12 :25 and 19:27 :21 resulted in the following:

$y = a.x + b = 0{,}0334x + 48{,}278$ $R^2 = 0{,}9995$

Here, every data-point for x represents 5 seconds. Therefore, the average slope a is equal to:

a = 0,0334 °C / 5 sec. = 6,68E-03 °C/second (which is 0,4 °C/minute)

Now solve where this curve hits the Tenv line:

[2] Measurements are recorded in HLT_open_loop_response_260404.xls

x = (47.41 – 48.278) / 6.68E-03 = -130 seconds (or 2 minutes and 10 seconds)
The dead-time moment is then 19:12:25 – 2:10 = 19:10:15
Therefore, the **dead-time Θ = 19:10:15 – 19:08:20 = 1:55 = 115 seconds**

The normalised slope a* is equal to :

$a^* = a / \Delta p$ = 6,68E-05 °C/(%.second)

4.2 Experiment 2: Determine gain K_{hlt} and time-constant τ_{hlt} of HLT-system

Manually set the PID-controller output (“gamma”) to a certain value (e.g. 20 %). The temperature starts to increase and follows the curve in figure 2.

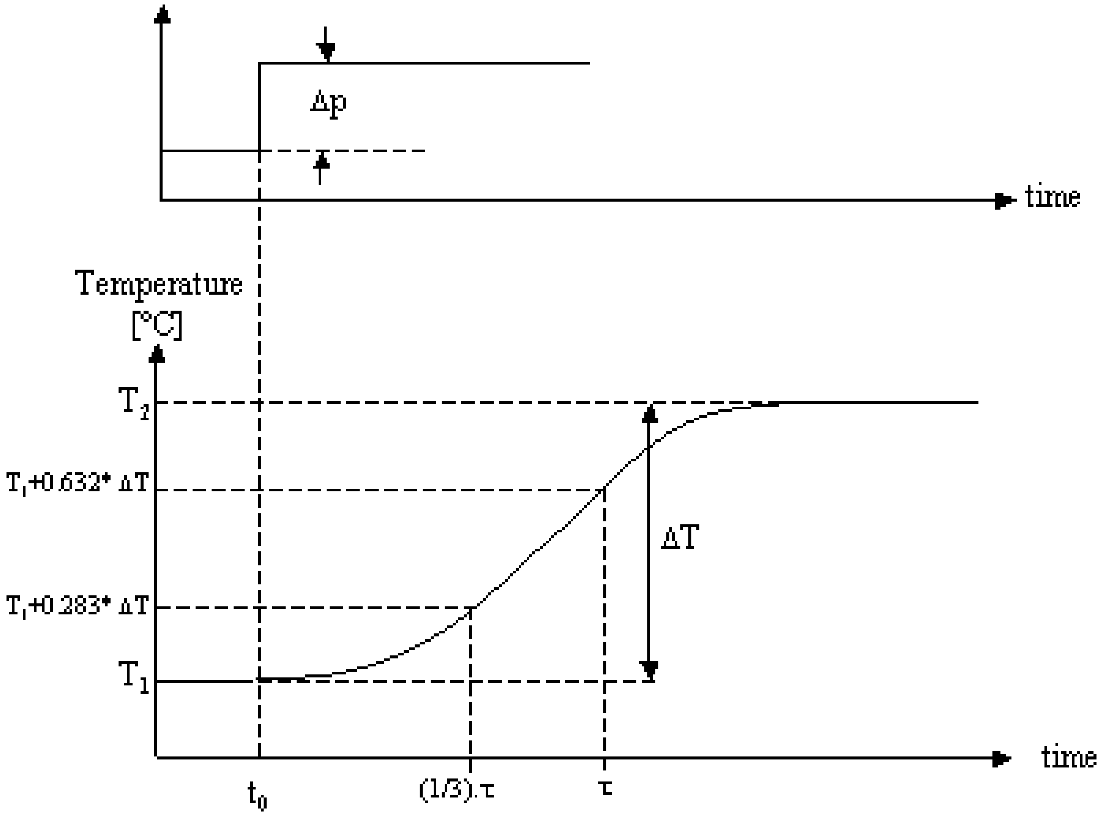


Figure 2: Step response of the system

Because of the large time-constant presumably present in the HLT system, the step response is not very accurate in determining the dead-time **Θ**. This is the mean reason to conduct two experiments (theoretically, the step-response would give you all the required information to determine the three parameters).

Experimental data:[3] HLT filled with 85 L water, lid off

09:49:04	t_0, the PID controller output gamma was set to 20 %, T_1 = 19,20 °C
20:24:03	Experiment stopped, T = 52,98 °C.
	Regression analysis (2nd order polynomial) used to find maximum.
	Maximum found at 21:20:44 (8300 ticks of 5 seconds). **T_2 = 52,989 °C**
	ΔT = 52,989 – 19,20 = 33,789 °C
	$T(t = 1/3.\tau_{hlt}) = T_1 + 0{,}283 * \Delta T$ = 28,762 °C => $1/3.\tau_{hlt}$ = 6599 seconds
	$T(t = \tau_{hlt}) = T_1 + 0{,}632 * \Delta T$ = 40,555 °C => τ_{hlt} = 16573 seconds
	Solve for τ_{hlt}: $\tau_{hlt} - 1/3.\,\tau_{hlt}$ = 16573 – 6599 => **τ_{hlt} = 14961 seconds**

Solve for Gain K_{hlt}: $K_{hlt} = \Delta T / \Delta p$ = 33,789 °C / 20 % = **1,689 °C / %**

[3] Measurements are recorded in HLT_step_response_290404.xls

4.3 Calculation of PID parameters for the various methods

In the previous paragraphs, the following parameters were found:

- **dead-time Θ** **= 115 seconds**
- **Gain K_{hlt}** **= 1,689 °C / %**
- **Time-constant τ_{hlt}** **= 14961 seconds**
- **Normalised slope a*** **= a / Δp = 6,68E-05 °C/(%.second)**

	Kc [%/°C]	Ti [sec]	Td [sec]	
Ziegler-Nichols Open Loop	156,2	230,0	57,5	PID
	117,2	383,0	- -	PI
Ziegler-Nichols Closed Loop	92,4	230,0	57,5	PID
	69,3	383,0	- -	PI
Cohen-Coon	102,8	282,2	41,8	PID
	69,4	377,2	- -	PI
ITAE-Load	80,8	489,0	44,9	PID
	59,2	810,2	- -	PI

Table 2: Calculation of parameters for the various methods

Referenced Documents

Referenced documents				
Version	**Date**	**Title:**	**Author**	**Tag**
	2005	"Digital Self-Tuning Controllers", ISBN 1-85233-980-2, Springer	V. Bobál, J. Böhm, J. Fessl, J. Machácek	[01]

Version History

Date	Version	Description
01-02-2003	V1.0	First version for display on web-site
08-03-2004	V2.0	- Removed implementation 1 and 3 - Added description of types A, B and C - Added type C PID algorithm - Added auto-tuning algorithm + example
10-03-2004	V2.1	Incorrect dead-time calculation
09-05-2004	V2.2	- Added Cohen-Coon + ITAE methods - Added two experiments + description - Calculations are updated
10-05-2004	V3.0	- Derivation with Taylor series replaced by exact derivation. Current implementation worked well, but simulation showed problems
13-05-2004	V3.01	- Update of document, corrected a few mistakes - Add low-pass filter to the D-term of the PID-controllers
13-05-2004	V3.02	Some textual changes
09-05-2006	V3.03	Error corrected in equation eq.09
26-02-2007	V3.10	- Referenced documents added - Typo corrected in eq.08 - Block diagram added - Source code shortened, non-relevant code removed - More consistent naming of variables - More elaborate derivation in chapter 2.
06-05-2011	V3.20	- Better derivation for pid_reg3() and pid_reg4(). Pid_reg3() is now a type A PID controller with filtering of the D-action. Pid_reg4() is now a pure Takahashi type C PID controller.

Appendix I: C-programs (pid_reg.h, pid_reg.c)

```
/*==================================================================
  File name    : $Id: pid_reg.h,v 1.7 2004/05/13 20:51:00 emile Exp $
  Author       : E. vd Logt
  ------------------------------------------------------------------
  Purpose : This file contains the defines for the PID controller.
  ------------------------------------------------------------------
  $Log: pid_reg.h,v $
  Revision 1.7  2004/05/13 20:51:00  emile
  V0.1 060302 First version
  ==================================================================
*/
#ifndef PID_REG_H
#define PID_REG_H

#ifdef __cplusplus
extern "C" {
#endif

// These defines are needed for loop timing and PID controller timing
#define TWENTY_SECONDS (400)
#define TEN_SECONDS    (200)
#define FIVE_SECONDS   (100)
#define ONE_SECOND      (20)
#define T_50MSEC  (50)   // Period time of TTimer in msec.

#define GMA_HLIM (100.0) // PID controller upper limit [%]
#define GMA_LLIM   (0.0) // PID controller lower limit [%]

typedef struct _pid_params
{
   double kc; // Controller gain from Dialog Box
   double ti; // Time-constant for I action from Dialog Box
   double td; // Time-constant for D action from Dialog Box
   double ts; // Sample time [sec.] from Dialog Box
   double k_lpf; // Time constant [sec.] for LPF filter
   double k0; // k0 value for PID controller
   double k1; // k1 value for PID controller
   double k2; // k2 value for PID controller
   double k3; // k3 value for PID controller
   double lpf1; // value for LPF filter
   double lpf2; // value for LPF filter
   int    ts_ticks;  // ticks for timer
   int    pid_model; // PID Controller type [0..3]
   double pp; // debug
   double pi; // debug
   double pd; // debug
} pid_params; // struct pid_params

//--------------------
// Function Prototypes
//--------------------
void init_pid2(pid_params *p);
void pid_reg2(double xk, double *yk, double tset, pid_params *p, int vrg);
void init_pid3(pid_params *p);
void pid_reg3(double xk, double *yk, double tset, pid_params *p, int vrg);
void init_pid4(pid_params *p);
void pid_reg4(double xk, double *yk, double tset, pid_params *p, int vrg);

#ifdef __cplusplus
};
#endif
#endif
```

```
/*==================================================================
  Function name: init_pid2(), pid_reg2()
                 init_pid3(), pid_reg3(), init_pid4(), pid_reg4()
  Author       : E. vd Logt
  File name    : $Id: pid_reg.c,v 1.6 2004/05/13 20:51:00 emile Exp $
  ------------------------------------------------------------------
  Purpose : This file contains the main body of the PID controller.
            For design details, please read the Word document
            "PID Controller Calculus".

            In the GUI, the following parameters can be changed:
            Kc: The controller gain
            Ti: Time-constant for the Integral Gain
            Td: Time-constant for the Derivative Gain
            Ts: The sample period [seconds]
  ------------------------------------------------------------------
  $Log: pid_reg.c,v $
  Revision 1.6  2004/05/13 20:51:00  emile
  ==================================================================
*/
#include "pid_reg.h"

static double ek_1;  // e[k-1]  = SP[k-1] - PV[k-1] = Tset_hlt[k-1] - Thlt[k-1]
static double ek_2;  // e[k-2]  = SP[k-2] - PV[k-2] = Tset_hlt[k-2] - Thlt[k-2]
static double xk_1;  // PV[k-1] = Thlt[k-1]
static double xk_2;  // PV[k-2] = Thlt[k-1]
static double yk_1;  // y[k-1]  = Gamma[k-1]
static double yk_2;  // y[k-2]  = Gamma[k-1]
static double lpf_1; // lpf[k-1] = LPF output[k-1]
static double lpf_2; // lpf[k-2] = LPF output[k-2]

void init_pid2(pid_params *p)
/*------------------------------------------------------------------
  Purpose  : This function initialises the PID controller, based on
             the new Type A PID controller.
  Variables: p: pointer to struct containing all PID parameters

                       Ts        2.Td
         k0 = Kc.(1 + ---- + ------------)
                      2.Ti   Ts + 2.k_lpf

                   Ts.Ts/Ti - 4.k_lpf - 4.Td
         k1 = Kc.( ------------------------- )
                          Ts + 2.k_lpf


                   2.k_lpf - Ts + Ts.Ts/(2.Ti) - k_lpf.Ts/Ti + 2.Td
         K2 = Kc.( ------------------------------------------------ )
                          Ts + 2.k_lpf


  Returns  : No values are returned
  ------------------------------------------------------------------*/
{
   double alfa = p->ts + 2.0 * p->k_lpf; // help variable

   p->ts_ticks = (int)((p->ts * 1000.0) / T_50MSEC);
   if (p->ts_ticks > TWENTY_SECONDS)
   {
      p->ts_ticks = TWENTY_SECONDS;
   }
   if (p->ti > 0.001)
   {
      p->k0  = p->kc * (+1.0 + (p->ts / (2.0 * p->ti))
                             + (2.0 * p->td / alfa));
      p->k1  = p->kc * (p->ts * p->ts / p->ti - 4.0 * p->k_lpf - 4.0 * p->td);
      p->k1 /= alfa;
      p->k2  = 2.0 * p->k_lpf - p->ts + p->ts * p->ts / (2.0 * p->ti);
      p->k2 += 2.0 * p->ti - p->k_lpf * p->ts / p->ti;
      p->k2 *= p->kc / alfa;
   } // if
   //--------------------------------------------------
   // u[k] = lpf1*u[k-1] + lpf2*u[k-2] + ....
   //--------------------------------------------------
   p->lpf1 = 4.0 * p->k_lpf / alfa;
   p->lpf2 = (p->ts - 2.0 * p->k_lpf) / alfa;
} // init_pid2()

void pid_reg2(double xk, double *yk, double tset, pid_params *p, int vrg)
/*------------------------------------------------------------------
  Purpose  : This function implements the updated PID controller.
             It is an update of pid_reg1(), derived with Bilinear
             Transformation. It is a Type A controller.
             This function should be called once every TS seconds.
  Variables:
        xk : The input variable x[k] (= measured temperature)
       *yk : The output variable y[k] (= gamma value for power electronics)
```

```
      tset : The setpoint value for the temperature
        *p : Pointer to struct containing PID parameters
        vrg: Release signal: 1 = Start control, 0 = disable PID controller
  Returns  : No values are returned
  ------------------------------------------------------------------*/
{
   double ek; // e[k]

   ek = tset - xk;  // calculate e[k]
   if (vrg)
   {
      *yk  = p->lpf1 * yk_1 + p->lpf2 * yk_2; // y[k] = p1*y[k-1] + p2*y[k-2]
      *yk += p->k0 * ek;                      //  ... + k0 * e[k]
      *yk += p->k1 * ek_1;                    //  ... + k1 * e[k-1]
      *yk += p->k2 * ek_2;                    //  ... + k2 * e[k-2]
   }
   else *yk = 0.0;

   ek_2 = ek_1; // e[k-2] = e[k-1]
   ek_1 = ek;   // e[k-1] = e[k]

   // limit y[k] to GMA_HLIM and GMA_LLIM
   if (*yk > GMA_HLIM)
   {
      *yk = GMA_HLIM;
   }
   else if (*yk < GMA_LLIM)
   {
      *yk = GMA_LLIM;
   } // else

   yk_2 = yk_1; // y[k-2] = y[k-1]
   yk_1 = *yk;  // y[k-1] = y[k]
} // pid_reg2()

void init_pid3(pid_params *p)
/*------------------------------------------------------------------
  Purpose  : This function initialises the Allen Bradley Type A PID
             controller.
  Variables: p: pointer to struct containing all PID parameters

                   Kc.Ts
             k0 =  -----   (for I-term)
                    Ti

                       Td
             k1 = Kc . --  (for D-term)
                       Ts

             The LPF parameters are also initialised here:
             lpf[k] = lpf1 * lpf[k-1] + lpf2 * lpf[k-2]
  Returns  : No values are returned
  ------------------------------------------------------------------*/
{
   p->ts_ticks = (int)((p->ts * 1000.0) / T_50MSEC);
   if (p->ts_ticks > TWENTY_SECONDS)
   {
      p->ts_ticks = TWENTY_SECONDS;
   }
   if (p->ti == 0.0)
   {
      p->k0 = 0.0;
   }
   else
   {
      p->k0 = p->kc * p->ts / p->ti;
   } // else
   p->k1   = p->kc * p->td / p->ts;
   p->lpf1 = (2.0 * p->k_lpf - p->ts) / (2.0 * p->k_lpf + p->ts);
   p->lpf2 = p->ts / (2.0 * p->k_lpf + p->ts);
} // init_pid3()

void pid_reg3(double xk, double *yk, double tset, pid_params *p, int vrg)
/*------------------------------------------------------------------
  Purpose  : This function implements the type Allen Bradley Type A PID
             controller. All terms are dependent on the error signal e[k].
             The D term is also low-pass filtered.
             This function should be called once every TS seconds.
  Variables:
        xk : The input variable x[k] (= measured temperature)
       *yk : The output variable y[k] (= gamma value for power electronics)
      tset : The setpoint value for the temperature
        *p : Pointer to struct containing PID parameters
        vrg: Release signal: 1 = Start control, 0 = disable PID controller
  Returns  : No values are returned
  ------------------------------------------------------------------*/
{
```

```
   double ek;  // e[k]
   double lpf; //LPF output

   ek = tset - xk;  // calculate e[k] = SP[k] - PV[k]
   //--------------------------------------
   // Calculate Lowpass Filter for D-term
   //--------------------------------------
   lpf = p->lpf1 * lpf_1 + p->lpf2 * (ek + ek_1);

   if (vrg)
   {
      //-----------------------------------------------------------
      // Calculate PID controller:
      // y[k] = y[k-1] + Kc*(e[k] - e[k-1] +
      //                     Ts*e[k]/Ti +
      //                     Td/Ts*(lpf[k] - 2*lpf[k-1]+lpf[k-2]))
      //-----------------------------------------------------------
      p->pp = p->kc * (ek - ek_1);  // y[k] = y[k-1] + Kc*(e[k] - e[k-1])
      p->pi = p->k0 * ek;           //      + Kc*Ts/Ti * e[k]
      p->pd = p->k1 * (lpf - 2.0 * lpf_1 + lpf_2);
      *yk += p->pp + p->pi + p->pd;
   }
   else *yk = 0.0;

   ek_1  = ek;    // e[k-1] = e[k]
   lpf_2 = lpf_1; // update stores for LPF
   lpf_1 = lpf;

   // limit y[k] to GMA_HLIM and GMA_LLIM
   if (*yk > GMA_HLIM)
   {
      *yk = GMA_HLIM;
   }
   else if (*yk < GMA_LLIM)
   {
      *yk = GMA_LLIM;
   } // else
} // pid_reg3()

void init_pid4(pid_params *p)
/*------------------------------------------------------------------
  Purpose  : This function initialises the Allen Bradley Type C PID
             controller.
  Variables: p: pointer to struct containing all PID parameters
  Returns  : No values are returned
  ------------------------------------------------------------------*/
{
   init_pid3(p); // identical to init_pid3()
} // init_pid4()

void pid_reg4(double xk, double *yk, double tset, pid_params *p, int vrg)
/*------------------------------------------------------------------
  Purpose  : This function implements the Takahashi PID controller,
             which is a type C controller: the P and D term are no
             longer dependent on the set-point, only on PV (which is Thlt).
             The D term is NOT low-pass filtered.
             This function should be called once every TS seconds.
  Variables:
        xk : The input variable x[k] (= measured temperature)
       *yk : The output variable y[k] (= gamma value for power electronics)
      tset : The setpoint value for the temperature
        *p : Pointer to struct containing PID parameters
        vrg: Release signal: 1 = Start control, 0 = disable PID controller
  Returns  : No values are returned
  ------------------------------------------------------------------*/
{
   double ek;  // e[k]
   double lpf; //LPF output

   ek = tset - xk;  // calculate e[k] = SP[k] - PV[k]

   if (vrg)
   {
      //-----------------------------------------------------------
      // Calculate PID controller:
      // y[k] = y[k-1] + Kc*(PV[k-1] - PV[k] +
      //                     Ts*e[k]/Ti +
      //                     Td/Ts*(2*PV[k-1] - PV[k] - PV[k-2]))
      //-----------------------------------------------------------
      p->pp = p->kc * (xk_1 - xk);  // y[k] = y[k-1] + Kc*(PV[k-1] - PV[k])
      p->pi = p->k0 * ek;           //      + Kc*Ts/Ti * e[k]
      p->pd = p->k1 * (2.0 * xk_1 - xk - xk_2);
      *yk += p->pp + p->pi + p->pd;
   }
   else { *yk = p->pp = p->pi = p->pd = 0.0; }

   xk_2  = xk_1;  // PV[k-2] = PV[k-1]
```

```
   xk_1  = xk;    // PV[k-1] = PV[k]

   // limit y[k] to GMA_HLIM and GMA_LLIM
   if (*yk > GMA_HLIM)
   {
      *yk = GMA_HLIM;
   }
   else if (*yk < GMA_LLIM)
   {
      *yk = GMA_LLIM;
   } // else
} // pid_reg4()
```